

MPI-PhT/94-30

TUM-T31-64/94

May 1994

CP Violation: Present and Future *

ANDRZEJ J. BURAS

*Technische Universität München, Physik Department
D-85748 Garching, Germany*

*Max-Planck-Institut für Physik
– Werner-Heisenberg-Institut –
Föhringer Ring 6, D-80805 München, Germany*

Abstract

We review the present status of CP violation in the standard model. Subsequently we make an excursion in the future in order to see what we could expect in this field in this and the next decade. We present various strategies for the determination of the CKM parameters and divide the decays into four classes with respect to theoretical uncertainties. We emphasize that the definitive tests of the Kobayashi-Maskawa picture of CP violation will come through a *simultaneous* study of CP asymmetries in $B_{d,s}^0$ decays, the rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$, and x_d/x_s . We illustrate how the measurements of the CP asymmetries in $B_{d,s}^0$ -decays together with a measurement of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ or $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and the known value of $|V_{us}|$ can determine *all* elements of the Cabibbo-Kobayashi-Maskawa matrix essentially without any hadronic uncertainties.

*Invited talk given at the First International Conference on Phenomenology of Unification from Present to Future, Rome - March 23-26, 1994, to appear in the proceedings.

Supported by the German Bundesministerium für Forschung und Technologie under contract 06 TM 732 and by the CEC science project SC1-CT91-0729.

CP Violation: Present and Future ^{*}

ANDRZEJ J. BURAS

*Technische Universität München, Physik Department
D-85748 Garching, Germany*

*Max-Planck-Institut für Physik
– Werner-Heisenberg-Institut –
Föhringer Ring 6, D-80805 München, Germany*

Abstract

We review the present status of CP violation in the standard model. Subsequently we make an excursion in the future in order to see what we could expect in this field in this and the next decade. We present various strategies for the determination of the CKM parameters and divide the decays into four classes with respect to theoretical uncertainties. We emphasize that the definitive tests of the Kobayashi-Maskawa picture of CP violation will come through a *simultaneous* study of CP asymmetries in $B_{d,s}^0$ decays, the rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$, and x_d/x_s . We illustrate how the measurements of the CP asymmetries in $B_{d,s}^0$ -decays together with a measurement of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ or $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and the known value of $|V_{us}|$ can determine *all* elements of the Cabibbo-Kobayashi-Maskawa matrix essentially without any hadronic uncertainties.

1 Setting the Scene

1.1 The Cabibbo-Kobayashi-Maskawa Matrix

In the Standard Model with three fermion generations, CP violation arises from a single phase in the unitary 3×3 Cabibbo-Kobayashi-Maskawa matrix [1, 2] which parametrizes the charged current interactions of quarks:

$$J_\mu^{cc} = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \quad (1)$$

^{*}Supported by the German Bundesministerium für Forschung und Technologie under contract 06 TM 732 and by the CEC science project SC1-CT91-0729.

Following Wolfenstein [3], it is useful but not necessary to expand each element of the CKM matrix as a power series in the small parameter $\lambda = |V_{us}| = 0.22$:

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \quad (2)$$

Because of the smallness of λ and the fact that for each element the expansion parameter is actually λ^2 , it is sufficient to keep only the first few terms in this expansion. Following [4] we will define the parameters $(\lambda, A, \varrho, \eta)$ through

$$s_{12} \equiv \lambda \quad s_{23} \equiv A\lambda^2 \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\varrho - i\eta) \quad (3)$$

where s_{ij} and δ enter the standard parametrization [5] of the CKM matrix. This specifies the higher orders terms in (2).

From tree level B decays sensitive to V_{cb} and V_{ub} , the parameters A , ϱ and η are constrained as follows [6]:

$$\lambda^2 A = |V_{cb}| = 0.038 \pm 0.004 \quad (4)$$

$$R_b \equiv \sqrt{\bar{\varrho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.36 \pm 0.09 \quad (5)$$

where we have introduced [4]

$$\bar{\varrho} = \varrho(1 - \frac{\lambda^2}{2}) \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}). \quad (6)$$

In order to determine ϱ and $|\eta|$ we still need the value of

$$R_t \equiv \sqrt{(1 - \bar{\varrho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right| \quad (7)$$

which is governed by $|V_{td}|$. From (5) and (7) we have $1 - R_b \leq R_t \leq 1 + R_b$ and unless $R_t = 1 \pm R_b$, one finds $\eta \neq 0$, which implies CP violation in the standard model.

We observe that within the standard model the measurements of four CP *conserving* decays sensitive to $|V_{us}|$, $|V_{ub}|$, $|V_{cb}|$ and $|V_{td}|$ can tell us whether CP violation is predicted in the standard model. This is a very remarkable property of the Kobayashi-Maskawa picture of CP violation: quark mixing and CP violation are closely related to each other. For this reason it is mandatory to discuss here also the most important CP conserving decays.

All this can be shown transparently in the $(\bar{\rho}, \bar{\eta})$ plane. Starting with the unitarity relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \quad (8)$$

rescaling it by $|V_{cd}V_{cb}^*| = A\lambda^3$ and depicting the result in the complex $(\bar{\rho}, \bar{\eta})$ plane, one finds the unitarity triangle of fig. 1. The lengths CB, CA and BA are equal to 1, R_b and R_t respectively. We observe that beyond the leading order in λ the point A does not correspond to (ϱ, η) but to $(\bar{\varrho}, \bar{\eta})$. Clearly within 3% accuracy $(\bar{\varrho}, \bar{\eta}) = (\varrho, \eta)$. In the distant future this difference may matter however.

Fig. 1

The triangle in fig. 1 is one of the important targets of the contemporary particle physics. Together with $|V_{us}|$ and $|V_{cb}|$ it summarizes the structure of the CKM matrix. In particular the area of the unrescaled triangle gives a measure of CP violation in the standard model [7]:

$$|J_{CP}| = 2 \cdot (\text{Area of } \Delta) = |V_{ud}| |V_{us}| |V_{ub}| |V_{cb}| \sin \delta = A^2 \lambda^6 \bar{\eta} = 0(10^{-5}). \quad (9)$$

This formula shows another important feature of the KM picture of CP violation: the smallness of CP violation in the standard model is not necessarily related to the smallness of η but to the fact that in this model the size of CP violating effects is given by products of small mixing parameters.

Since the top quark mass is an important parameter in the field of CP violation, we have to specify what we mean by m_t . Here in accordance with various QCD calculations quoted below, we will use $m_t \equiv \overline{m}_t(m_t)$: the current top quark mass at the scale m_t . The physical top quark mass (m_t^{phys}) defined as the pole of the renormalized propagator is by about 7 GeV higher than m_t .

Finally it should be stated that a large part in the errors quoted in (4) and (5) results from theoretical uncertainties. Consequently even if the data from CLEO improves in the future, it is difficult to imagine at present that in the tree level B-decays a better accuracy than $\Delta |V_{cb}| = \pm 2 \cdot 10^{-3}$ and $\Delta |V_{ub}/V_{cb}| = \pm 0.01$ ($\Delta R_b = \pm 0.04$) could be achieved [8]. We will see below that the loop induced decays governed by short distance physics can in principle offer a more accurate determination of $|V_{cb}|$ and $|V_{ub}/V_{cb}|$.

1.2 Loop induced Decays and Transitions

Using (4), (5) and (7) we find $|V_{td}| \leq A\lambda^3(1+R_b) \leq 13.4 \cdot 10^{-3}$ and the branching ratio $Br(t \rightarrow d) \leq 10^{-3}$. Consequently it will be very difficult to measure $|V_{td}|$ in tree level top quark decays. In order to find $|V_{td}|$ we have to measure loop induced decays and transitions governed by penguin and box diagrams with internal top quark exchanges.

In the *K-meson* system the top favourites are: the indirect (ε_K) and the direct (ε') CP violating contributions to $K \rightarrow \pi\pi$, the rare decays $K_L \rightarrow \pi^0 e^+ e^-$, $K_L \rightarrow \mu\bar{\mu}$, $K^+ \rightarrow \pi^+ \nu\bar{\nu}$, $K_L \rightarrow \pi^0 \nu\bar{\nu}$ and the parity violating asymmetry Δ_{LR} in $K^+ \rightarrow \pi^+ \mu^+ \mu^-$.

In the *B-meson* system the corresponding favourites are: $B_d^0 - \bar{B}_d^0$ ($B_s^0 - \bar{B}_s^0$) mixing described by the parameter x_d (x_s) and the rare decays $B \rightarrow \mu\bar{\mu}$, $B \rightarrow X_{d,s} \nu\bar{\nu}$, $B \rightarrow X_{d,s} \gamma$ and $B \rightarrow X_{d,s} e^+ e^-$.

Furthermore a very special role is played by CP-asymmetries in the decays $B_{d,s}^0 \rightarrow f$ where f is a CP eigenstate. Some of these asymmetries determine the angles in fig.1 (α , β , γ) without any theoretical uncertainties [9]. Consequently their measurements will have important impact on the search of the unitarity triangle (Δ) and indirectly on $|V_{td}|$. We will return to CP asymmetries in sections 5-7.

From this long list only ε_K and x_d are useful for Δ at present but in 15 years from now the picture of Δ might well look like the one shown in fig.2.

The general structure of theoretical expressions for the relevant decay amplitudes is given in a simplified form roughly as follows:

$$A(\text{Decay}) = BV_{CKM}\eta_{QCD}F(m_t) + (\text{Charm Contributions}) + (\text{LD Contributions}) \quad (10)$$

Here V_{CKM} represents a given product of the CKM elements we want to determine. $F(m_t)$ results from the evaluation of loop diagrams with top exchanges and η_{QCD} summarizes short distance QCD corrections to a given decay. By now these corrections are known essentially for all decays listed above at the leading and next-to-leading order in the renormalization group improved perturbation theory. Next B stands for a non-perturbative factor related to the relevant hadronic matrix element of the contributing four fermion operator: the main theoretical uncertainty in the whole enterprise. In semi-leptonic decays such as $K \rightarrow \pi \nu\bar{\nu}$, the non-perturbative B -factors can fortunately be determined from leading tree level decays such as $K^+ \rightarrow \pi^0 e^+ \nu$ reducing or removing the non-perturbative uncertainty. In non-leptonic decays and in $B^0 - \bar{B}^0$ mixing this is generally not possible and we have to rely on existing non-perturbative methods. The additional terms in (10) include internal charm contributions and sometimes unwanted long distance contributions as in the case of $K_L \rightarrow \mu\bar{\mu}$. In B-decays the internal top

contributions are essentially the whole story. In K-decays the internal charm contributions can sometimes be also important as in the case of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Finally in more complicated decays, in particular in ε' , one finds linear combinations of different m_t -dependent functions. Moreover due to the appearance of several contributing operators a set of different B-factors can be present.

Fig. 2

1.3 Classification

Let us group the various decays and quantities in four different classes with respect to hadronic uncertainties present in them.

- Class I (Essentially no hadronic uncertainties):
 $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and some CP asymmetries in neutral B decays to CP eigenstates which give $\sin 2\alpha$, $\sin 2\beta$, $\sin 2\gamma$.
- Class II (Small theoretical uncertainties related to $\Lambda_{\overline{MS}}$, m_c , the renormalization scale μ or $SU(3)$ -breaking effects.):
 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, the parity violating asymmetry Δ_{LR} , x_d/x_s and $B \rightarrow X_{d,s} \nu \bar{\nu}$.
- Class III (Hadronic uncertainties are present but will probably be reduced con-

siderably in the next five years):

$$K_L \rightarrow \pi^0 e^+ e^-, \varepsilon_K, x_{d,s}, B_{d,s} \rightarrow \mu \bar{\mu}, B \rightarrow X_{d,s} \gamma, B \rightarrow X_{d,s} e^+ e^-.$$

- Class IV (Large hadronic uncertainties which can only be removed if some dramatic improvements in non-perturbative techniques will take place):

$$\varepsilon', K_L \rightarrow \mu \bar{\mu}, \text{CP asymmetries in } B^\pm \text{ and hyperon decays.}$$

It should be stressed that all these decays are very interesting. In particular, in addition to $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in class I, also $\text{Re}(\varepsilon'/\varepsilon)$ and B^+ decays in class IV may give first signals of the direct CP violation. However only the decays of class I and II, when measured, allow a reliable determination of CKM parameters unless considerable improvements in non-perturbative techniques will be made. The B -factors and long distance contributions in class III are easier to calculate than for quantities in class IV.

2 Strategy

During the coming fifteen years we will certainly witness a dramatic improvement in the determination of the CKM-parameters analogous to, although not as precise as, the determination of the parameters in the gauge boson sector which took place during the last years. Let us recall that the relevant *independent* parameters in the electroweak precision studies are:

$$G_F, \quad \alpha_{QED}(m_e), \quad M_Z, \quad m_t, \quad m_H \quad (11)$$

with α_{QCD} or $\Lambda_{\overline{MS}}$ playing sometimes some role. In the field of quark mixing and CP violation the relevant parameters are

$$\lambda, \quad A, \quad \varrho, \quad \eta, \quad m_t \quad (12)$$

with $\Lambda_{\overline{MS}}$ and m_c playing often sizable roles. Moreover as stated above, non-perturbative effects in class III and IV decays play a very important role. On the other hand due to the natural flavour conservation in neutral current processes and the small couplings of the neutral higgs to s and b quarks, the impact of m_H on this field can be fully neglected.

There is of course most probably and hopefully some new physics beyond the standard model. This physics introduces generally new parameters and makes the full discussion considerably more involved. Moreover theoretical calculations, in particular of the QCD corrections, for the extensions of the standard model are often not at the

level of existing calculations in this model and consequently no precise discussion of various effects related to new physics is possible at present.

Our strategy then will be to confine the presentation exclusively to the standard model and to discuss several quantities simultaneously with the hope that future precise measurements will display some inconsistencies which will signal a new physics beyond the standard model. Moreover we will devote a large part of this review to decays of class I and II which being essentially free from hadronic uncertainties are ideally suited for the determination of the CKM parameters. We will however also discuss some of the decays of class III and IV.

3 Messages from the Indirect CP Violation

The indirect CP violation in the K-system discovered in 1964 [10] and parametrized by ε_K is the only clear experimental signal of this important phenomenon. The usual box diagram calculation together with the experimental value $\varepsilon_K = 2.26 \cdot 10^{-3} \exp(i\pi/4)$ specifies a hyperbola in the $(\bar{\rho}, \bar{\eta})$ plane with $\bar{\eta} > 0$ as shown in fig.2. The position of this hyperbola depends on m_t , V_{cb} and on the non-perturbative parameter B_K . There are essentially four messages here:

- For a given set $(m_t, V_{cb}, |V_{ub}/V_{cb}|, B_K)$ one determines two values of η corresponding to two crossing points of the ϵ_K -hyperbola with the circle (5). Typically $0.20 < \eta < 0.45$ is found.
- With decreasing $|V_{cb}|$, B_K and m_t , the ϵ_K -hyperbola moves away from the origin of the $(\bar{\rho}, \bar{\eta})$ plane. When the hyperbola and the circle (5) only touch each other a lower bound for m_t follows [11]:

$$(m_t)_{min} = M_W \left[\frac{1}{2A^2} \left(\frac{1}{A^2 B_K R_b} - 1.2 \right) \right]^{0.658} \quad (13)$$

For $V_{cb} = 0.040$, $|V_{ub}/V_{cb}| = 0.08$ and $B_K = 0.75$ one has $(m_t)_{min} = 170 \text{ GeV}$.

- For a given m_t a lower bound on $|V_{cb}|$, $|V_{ub}/V_{cb}|$ and B_K can be found. For instance [4]

$$(B_K)_{min} = \left[A^2 R_b \left(2x_t^{0.76} A^2 + 1.2 \right) \right]^{-1}; \quad x_t = \frac{m_t^2}{M_W^2} \quad (14)$$

- The CDF value for m_t ($m_t^{phys} = 174 \pm 16 \text{ GeV}$) [12] together with (13) and (14) imply that the observed indirect CP violation can be accommodated in the standard

model provided $|V_{cb}|$, $|V_{ub}/V_{cb}|$ and B_K are not too small. For instance with $m_t < 180 \text{ GeV}$, $|V_{ub}/V_{cb}| < 0.09$ and $|V_{cb}| < 0.040$ only values $B_K > 0.62$ are allowed. Such values are found in the lattice ($B_K = 0.83 \pm 0.03$) [13] and the $1/N$ approach ($B_K = 0.7 \pm 0.1$) [14].

To summarize: the presently measured values of $|V_{ub}/V_{cb}|$, $|V_{cb}|$ and m_t together with the non-perturbative calculations of B_K imply that the KM picture of CP violation is consistent with the bound in (13). In view of large uncertainties in the four input parameters in question this first test of the KM picture is however by no means conclusive.

4 ε'/ε , $K_L \rightarrow \pi^\circ e^+ e^-$, $K_L \rightarrow \pi^\circ \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Let me now discuss four stars in the field of K-decays. The first three deal with searches of direct CP violation. The last one is CP conserving but plays an important role in the determination of the unitarity triangle. The fifth star, the parity violating asymmetry in $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ (Δ_{LR}) [15] is discussed elsewhere [16].

4.1 ε'/ε

$\text{Re}(\varepsilon'/\varepsilon)$ measures the ratio of direct to indirect CP violation in $K \rightarrow \pi\pi$ decays. In the standard model ε'/ε is governed by QCD penguins and electroweak (EW) penguins [17]. In spite of being suppressed by α/α_s relative to QCD penguin contributions, the electroweak penguin contributions have to be included because of the additional enhancement factor $\text{Re}A_0/\text{Re}A_2 = 22$ relative to QCD penguins. Moreover with increasing m_t the EW-penguins become increasingly important [18, 19] and entering ε'/ε with the opposite sign to QCD-penguins suppress this ratio for large m_t . For $m_t \approx 200 \text{ GeV}$ the ratio can even be zero [19]. The short distance QCD corrections to ε'/ε are known at the NLO level [20, 21]. Unfortunately ε'/ε is plagued with uncertainties related to non-perturbative B-factors which multiply m_t dependent functions in a formula like (10). Several of these B-factors can be extracted from leading CP-conserving $K \rightarrow \pi\pi$ decays [20]. Two important B-factors (B_6 = the dominant QCD penguin and B_8 = the dominant electroweak penguin) cannot be determined this way and one has to use lattice or $1/N$ methods to predict $\text{Re}(\varepsilon'/\varepsilon)$. An analytic formula for $\text{Re}(\varepsilon'/\varepsilon)$ as a function of m_t , $\Lambda_{\overline{MS}}$, B_6 , B_8 , m_s and V_{CKM} can be found in [22]. A very simplified version of

this formula is given as follows

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = 12 \cdot 10^{-4} \left[\frac{\eta \lambda^5 A^2}{1.7 \cdot 10^{-4}} \right] \left[\frac{150 \text{ MeV}}{m_s(m_c)} \right]^2 [B_6 - Z(x_t)B_8] \quad (15)$$

where $Z(x_t)$ is given in (20). For $m_t = 170 \pm 10 \text{ GeV}$ and using ε_K -analysis to determine η one finds [20, 21]

$$2 \cdot 10^{-4} \leq \frac{\varepsilon'}{\varepsilon} \leq 1 \cdot 10^{-3} \quad (16)$$

if $B_6 \approx B_8 \approx 1$ (lattice, $1/N$ expansion) are used. For $B_6 \approx 2$ and $B_8 \approx 1$ as advocated in [23], $\text{Re}(\varepsilon'/\varepsilon)$ increases to $(15 \pm 5) \cdot 10^{-4}$.

The experimental situation on $\text{Re}(\varepsilon'/\varepsilon)$ is unclear at present. While the result of NA31 collaboration at CERN with $\text{Re}(\varepsilon'/\varepsilon) = (23 \pm 7) \cdot 10^{-4}$ [24] clearly indicates direct CP violation, the value of E731 at Fermilab, $\text{Re}(\varepsilon'/\varepsilon) = (7.4 \pm 5.9) \cdot 10^{-4}$ [25] is compatible with superweak theories [26] in which $\varepsilon'/\varepsilon = 0$. Both results are in the ballpark of the theoretical estimates although the NA31 result appears a bit high compared to the range given in (16).

Hopefully, in about five years the experimental situation concerning ε'/ε will be clarified through the improved measurements by the two collaborations at the 10^{-4} level and by experiments at the Φ factory in Frascati. One should also hope that the theoretical situation of ε'/ε will improve by then to confront the new data.

4.2 $K_L \rightarrow \pi^0 e^+ e^-$

Whereas in $K \rightarrow \pi\pi$ decays the CP violating contribution is a tiny part of the full amplitude and the direct CP violation is expected to be at least by three orders of magnitude smaller than the indirect CP violation, the corresponding hierarchies are very different for the rare decay $K_L \rightarrow \pi^0 e^+ e^-$. At lowest order in electroweak interactions (single photon, single Z-boson or double W-boson exchange), this decay takes place only if CP symmetry is violated [27]. Moreover, the direct CP violating contribution is predicted to be larger than the indirect one. The CP conserving contribution to the amplitude comes from a two photon exchange, which although higher order in α could in principle be sizable. The studies of the last years [28] indicate however that the CP conserving part is significantly smaller than the direct CP violating contribution.

The size of the indirect CP violating contribution will be known once the CP conserving decay $K_S \rightarrow \pi^0 e^+ e^-$ has been measured [29]. On the other hand the direct CP violating contribution can be fully calculated as a function of m_t , CKM parameters and the QCD coupling constant α_s . There are practically no theoretical uncertainties related to hadronic matrix elements in this part, because the latter can be extracted

from the well-measured decay $K^+ \rightarrow \pi^0 e^+ \nu$. The next-to-leading QCD corrections to the direct CP violating contribution have been recently calculated [30] reducing certain ambiguities present in leading order analyses [31] and enhancing the leading order results by roughly 25%. The final result is given by

$$Br(K_L \rightarrow \pi^0 e^+ e^-)_{dir} = 4.16 \cdot (Im\lambda_t)^2 (y_{7A}^2 + y_{7V}^2) \quad (17)$$

where $Im\lambda_t = Im(V_{td}V_{ts}^*)$ and

$$y_{7V} = \frac{\alpha}{2\pi \sin^2 \theta_W} (P_0 + Y(x_t) - 4 \sin^2 \theta_W Z(x_t)), \quad (18)$$

$$y_{7A} = -\frac{\alpha}{2\pi \sin^2 \theta_W} Y(x_t). \quad (19)$$

Here, to a very good approximation for $140 \text{ GeV} \leq m_t \leq 230 \text{ GeV}$,

$$Y(x_t) = 0.315 \cdot x_t^{0.78}, \quad Z(x_t) = 0.175 \cdot x_t^{0.93}. \quad (20)$$

Next $P_0 = 0.70 \pm 0.02$ as found in [30]. For $m_t = 170 \pm 10 \text{ GeV}$ one finds

$$Br(K_L \rightarrow \pi^0 e^+ e^-)_{dir} = (5. \pm 2.) \cdot 10^{-12} \quad (21)$$

where the error comes dominantly from the uncertainties in the CKM parameters. This should be compared with the present estimates of the other two contributions: $Br(K_L \rightarrow \pi^0 e^+ e^-)_{indir} \leq 1.6 \cdot 10^{-12}$ and $Br(K_L \rightarrow \pi^0 e^+ e^-)_{cons} \approx (0.3 - 1.8) \cdot 10^{-12}$ for the indirect CP violating and the CP conserving contributions respectively [28]. Thus direct CP violation is expected to dominate this decay.

The present experimental bounds

$$Br(K_L \rightarrow \pi^0 e^+ e^-) \leq \begin{cases} 4.3 \cdot 10^{-9} & [32] \\ 5.5 \cdot 10^{-9} & [33] \end{cases} \quad (22)$$

are still by three orders of magnitude away from the theoretical expectations in the Standard Model. Yet the prospects of getting the required sensitivity of order 10^{-11} – 10^{-12} in six years are encouraging [34].

4.3 $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ are the theoretically cleanest decays in the field of rare K-decays. $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is dominated by short distance loop diagrams involving the top quark and proceeds almost entirely through direct CP violation [35]. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is CP conserving and receives contributions from both internal top and charm exchanges. The last year calculations [36, 37] of next-to-leading QCD corrections to

these decays considerably reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expressions [38]. Since the relevant hadronic matrix elements of the weak current $\bar{s}\gamma_\mu(1 - \gamma_5)d$ can be measured in the leading decay $K^+ \rightarrow \pi^0 e^+ \nu$, the resulting theoretical expressions for $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ are only functions of the CKM parameters, the QCD scale $\Lambda_{\overline{MS}}$ and the quark masses m_t and m_c . The long distance contributions to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ have been considered in [39] and found to be very small: two to three orders of magnitude smaller than the short distance contribution at the level of the branching ratio. The long distance contributions to $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are negligible as well.

The explicit expressions for $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ are given as follows

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 4.64 \cdot 10^{-11} \cdot \left[\left(\frac{\text{Im}\lambda_t}{\lambda^5} X(x_t) \right)^2 + \left(\frac{\text{Re}\lambda_c}{\lambda} P_0(K^+) + \frac{\text{Re}\lambda_t}{\lambda^5} X(x_t) \right)^2 \right] \quad (23)$$

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 1.94 \cdot 10^{-10} \cdot \left(\frac{\text{Im}\lambda_t}{\lambda^5} X(x_t) \right)^2 \quad (24)$$

λ_c is essentially real and $X(x_t)$ is given to an excellent accuracy by

$$X(x_t) = 0.65 \cdot x_t^{0.575} \quad (25)$$

where the NLO correction calculated in [36] is included if $m_t \equiv \bar{m}_t(m_t)$. Next $P_0(K^+) = 0.40 \pm 0.09$ [37, 41] is a function of m_c and $\Lambda_{\overline{MS}}$ and includes the residual uncertainty due to the renormalization scale μ . The absence of P_0 in (24) makes $K_L \rightarrow \pi^0 \nu \bar{\nu}$ theoretically even cleaner than $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

It has been pointed out in [40] that measurements of $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ could determine the unitarity triangle completely provided m_t and V_{cb} are known. Generalizing this analysis to include non-leading terms in λ one finds to a very good accuracy [41] ($\sigma = (1 - \lambda^2/2)^{-2}$):

$$\bar{\varrho} = 1 + \frac{P_0(K^+) - \sqrt{\sigma(B_+ - B_L)}}{A^2 X(x_t)} \quad \bar{\eta} = \frac{\sqrt{B_L}}{\sqrt{\sigma} A^2 X(x_t)} \quad (26)$$

where we have introduced the "reduced" branching ratios

$$B_+ = \frac{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{4.64 \cdot 10^{-11}} \quad B_L = \frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{1.94 \cdot 10^{-10}} \quad (27)$$

It follows from (26) that

$$r_s(B_+, B_L) = \frac{1 - \bar{\varrho}}{\bar{\eta}} = \cot \beta = \sqrt{\sigma} \frac{\sqrt{\sigma(B_+ - B_L)} - P_0(K^+)}{\sqrt{B_L}} \quad (28)$$

so that

$$\sin 2\beta = \frac{2r_s(B_+, B_L)}{1 + r_s^2(B_+, B_L)} \quad (29)$$

does not depend on m_t and V_{cb} . An exact treatment of the CKM matrix confirms this finding to high accuracy. Consequently $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ offer a clean determination of $\sin 2\beta$ which can be confronted with the one possible in $B^0 \rightarrow \psi K_S$ discussed below. Combining these two ways of determining $\sin 2\beta$ one finds an interesting relation between rare K decays and B physics

$$\frac{2r_s(B_+, B_L)}{1 + r_s^2(B_+, B_L)} = -A_{CP}(\psi K_S) \frac{1 + x_d^2}{x_d} \quad (30)$$

which must be satisfied in the standard model. Any deviation from this relation would signal new physics. A numerical analysis of (23), (24) and (29) will be given below.

5 CP Asymmetries in B-Decays and x_d/x_s

5.1 CP-Asymmetries in B^0 -Decays

The CP-asymmetry in the decay $B_d^0 \rightarrow \psi K_S$ allows in the standard model a direct measurement of the angle β in the unitarity triangle without any theoretical uncertainties [9]. Similarly the decay $B_d^0 \rightarrow \pi^+ \pi^-$ gives the angle α , although in this case strategies involving other channels are necessary in order to remove hadronic uncertainties related to penguin contributions [42]. The determination of the angle γ from CP asymmetries in neutral B-decays is more difficult but not impossible [43]. Also charged B decays could be useful in this respect [44]. We have for instance

$$A_{CP}(\psi K_S) = -\sin(2\beta) \frac{x_d}{1 + x_d^2}, \quad A_{CP}(\pi^+ \pi^-) = -\sin(2\alpha) \frac{x_d}{1 + x_d^2} \quad (31)$$

where we have neglected QCD penguins in $A_{CP}(\pi^+ \pi^-)$. Since in the triangle of fig.1 one side is known, it suffices to measure two angles to determine the triangle completely. We will investigate the impact of the future measurements of $\sin 2\alpha$ and $\sin 2\beta$ below. $\sin(2\phi_i)$ can be expressed in terms of $(\bar{\varrho}, \bar{\eta})$ as follows [4]

$$\sin(2\alpha) = \frac{2\bar{\eta}(\bar{\eta}^2 + \bar{\varrho}^2 - \bar{\varrho})}{(\bar{\varrho}^2 + \bar{\eta}^2)((1 - \bar{\varrho})^2 + \bar{\eta}^2)} \quad \sin(2\beta) = \frac{2\bar{\eta}(1 - \bar{\varrho})}{(1 - \bar{\varrho})^2 + \bar{\eta}^2} \quad (32)$$

We will see below that the asymmetry $A_{CP}(\psi K_S)$ could be as high as -0.4 . This is not in contradiction with (9) because the corresponding branching ratio for this decay is $O(10^{-4})$. This possibility of observing large CP asymmetries in B-decays makes them particularly useful for the tests of the KM picture.

5.2 $B^o - \bar{B}^o$ Mixing

Measurement of $B_d^o - \bar{B}_d^o$ mixing parametrized by x_d allows to determine R_t :

$$R_t = 1.63 \cdot \frac{R_0}{\sqrt{S(x_t)}} \quad S(x_t) = 0.784 \cdot x_t^{0.76} \quad (33)$$

where

$$R_0 \equiv \sqrt{\frac{x_d}{0.72}} \left[\frac{200 \text{ MeV}}{F_{B_d} \sqrt{B_{B_d}}} \right] \left[\frac{0.038}{\kappa} \right] \sqrt{\frac{0.55}{\eta_B}} \quad \kappa \equiv |V_{cb}| \left[\frac{\tau_B}{1.5 \text{ ps}} \right]^{0.5} \quad (34)$$

with τ_B being the B-meson life-time. $\eta_B = 0.55$ is the QCD factor [45]. F_{B_d} is the B-meson decay constant and B_{B_d} denotes a non-perturbative parameter analogous to B_K . The values of x_d , $F_{B_d} \sqrt{B_{B_d}}$ and $|V_{cb}|$ will be specified below.

The accuracy of the determination of R_t can be considerably improved by measuring simultaneously the $B_s^o - \bar{B}_s^o$ mixing described by x_s . We have

$$R_t = \frac{1}{\sqrt{R_{ds}}} \sqrt{\frac{x_d}{x_s}} \frac{1}{\lambda} \sqrt{1 - \lambda^2(1 - 2\varrho)} \quad R_{ds} = \frac{\tau_{B_d}}{\tau_{B_s}} \cdot \frac{m_{B_d}}{m_{B_s}} \left[\frac{F_{B_d} \sqrt{B_{B_d}}}{F_{B_s} \sqrt{B_{B_s}}} \right]^2 \quad (35)$$

Note that m_t has been eliminated this way and R_{ds} depends only on SU(3)-flavour breaking effects which contain much smaller theoretical uncertainties than the hadronic matrix elements in x_d and x_s separately. An estimate of such effects in $F_{B_d} \sqrt{B_{B_d}} / F_{B_s} \sqrt{B_{B_s}}$ [46] shows that provided x_d/x_s has been accurately measured a determination of R_t within $\pm 10\%$ should be possible. We will soon see that a much more accurate determination of R_t can be achieved by measuring CP asymmetries in B-decays.

6 Future Visions

Here I would like to report on the results of recent studies presented in detail in refs. [4, 41, 47]. After showing the present picture of the unitarity triangle corresponding to the range of parameters given in (36) we will investigate what the future could bring us in this field. Several lines of attack will be presented in this section culminating with a precise determination of all CKM parameters in section 7.

6.1 ε_K , $B_d^o - \bar{B}_d^o$, $|V_{ub}/V_{cb}|$, $|V_{cb}|$

Here we just use the four quantities listed above anticipating improved determinations of m_t , $|V_{ub}/V_{cb}|$, B_K , $F_B \sqrt{B_B}$ and x_d in the next ten years (ranges II and III). The measurements by CLEO and at LEP will play important roles here. In view of our remarks in section 1.1, the range III assumes also improvements in the theory. We consider the following ranges [4]:

Range I

$$\begin{aligned}
|V_{cb}| &= 0.038 \pm 0.004 & |V_{ub}/V_{cb}| &= 0.08 \pm 0.02 \\
B_K &= 0.7 \pm 0.2 & \sqrt{B_{B_d}} F_{B_d} &= (200 \pm 30) \text{ MeV} \\
x_d &= 0.72 \pm 0.08 & m_t &= (165 \pm 15) \text{ GeV}
\end{aligned} \tag{36}$$

Range II

$$\begin{aligned}
|V_{cb}| &= 0.040 \pm 0.002 & |V_{ub}/V_{cb}| &= 0.08 \pm 0.01 \\
B_K &= 0.75 \pm 0.07 & \sqrt{B_{B_d}} F_{B_d} &= (185 \pm 15) \text{ MeV} \\
x_d &= 0.72 \pm 0.04 & m_t &= (170 \pm 7) \text{ GeV}
\end{aligned} \tag{37}$$

Range III

$$\begin{aligned}
|V_{cb}| &= 0.040 \pm 0.001 & |V_{ub}/V_{cb}| &= 0.08 \pm 0.005 \\
B_K &= 0.75 \pm 0.05 & \sqrt{B_{B_d}} F_{B_d} &= (185 \pm 10) \text{ MeV} \\
x_d &= 0.72 \pm 0.04 & m_t &= (170 \pm 5) \text{ GeV}
\end{aligned} \tag{38}$$

The resulting unitarity triangles for ranges I-III are shown in the left half of fig. 3. For the range III one has the following expectations:

$$\begin{aligned}
\sin 2\alpha &= 0.50 \pm 0.49 & \sin 2\beta &= 0.61 \pm 0.09 \\
|V_{td}| &= (9.4 \pm 1.0) \cdot 10^{-3} & x_s &= 12.9 \pm 2.8 \\
Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) &= (1.03 \pm 0.15) \cdot 10^{-10} & Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= (2.7 \pm 0.4) \cdot 10^{-11}
\end{aligned} \tag{39}$$

and $\sin(2\gamma) = 0. \pm 0.68$. We should remark that for the ranges II and III, the uncertainties in $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ due to m_c , $\Lambda_{\overline{MS}}$ and μ have been omitted. They will be included in sections 6.2 and 7.

This exercise implies that if the accuracy of various parameters given in (37) and (38) is achieved, the determination of $|V_{td}|$ and the predictions for $\sin(2\beta)$, $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ are quite accurate. A sizable uncertainty in x_s remains however. Another important message from this analysis is the inability of a precise determination of $\sin(2\alpha)$ and $\sin(2\gamma)$ on the basis of ε_K , $B^0 - \bar{B}^0$, $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ alone. This analysis shows that even with the improved values of the parameters in question as given in (37) and (38) a precise determination of $\sin(2\alpha)$ and $\sin(2\gamma)$ this way should not be expected.

6.2 $\sin(2\beta)$ from $K \rightarrow \pi \nu \bar{\nu}$

The numerical analysis of (26)–(29) shows [41] that provided $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ are measured within $\pm 10\%$ accuracy, $\Delta \sin 2\beta = \pm 0.11$ could be achieved this way. With decreasing uncertainty in $\Lambda_{\overline{MS}}$ and m_c this error could be

reduced to $\Delta \sin 2\beta < \pm 0.10$. The determination of $\sin 2\alpha$ and $\sin 2\gamma$ on the other hand is rather poor. However respectable determinations of the Wolfenstein parameter η and of $|V_{td}|$ can be obtained. Choosing $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10}$, $Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (2.5 \pm 0.25) \cdot 10^{-11}$, $m_t = (170 \pm 5) GeV$ and $|V_{cb}| = 0.040 \pm 0.001$ one finds [41]

$$\sin(2\beta) = 0.60 \pm 0.11 \quad \eta = 0.34 \pm 0.05 \quad |V_{td}| = (9.3 \pm 2.1) \cdot 10^{-10} \quad (40)$$

Fig. 3

6.3 CP Asymmetries in B^0 -Decays

Let us next investigate the impact of the measurements of $\sin(2\alpha)$ and $\sin(2\beta)$ on the determination of the unitarity triangle. As a warming up let us consider the accuracies

$\Delta \sin(2\beta) = \pm 0.06$ and $\Delta \sin(2\alpha) = \pm 0.10$ which could be achieved around the year 2000 [48, 49].

In the right half of fig. 3 [4] we show the impact of such measurements taking

$$\sin(2\beta) = \begin{cases} 0.60 \pm 0.18 & \text{(a)} \\ 0.60 \pm 0.06 & \text{(b)} \end{cases} \quad \sin(2\alpha) = \begin{cases} -0.20 \pm 0.10 & \text{(I)} \\ 0.10 \pm 0.10 & \text{(II)} \\ 0.70 \pm 0.10 & \text{(III)} \end{cases} \quad (41)$$

We observe that the measurement of $\sin(2\alpha)$ in conjunction with $\sin(2\beta)$ at the expected precision will have a large impact on the accuracy of the determination of the unitarity triangle and of the CKM parameters. In order to show this more explicitly we take $\sin(2\beta) = 0.60 \pm 0.06$, $\sin(2\alpha) = 0.10 \pm 0.10$ and the values of $|V_{cb}|$, x_d and m_t of the vision (37) to find [4]

$$\begin{aligned} |V_{td}| &= (8.8 \pm 0.4) \cdot 10^{-3} & x_s &= 16.3 \pm 1.3 \\ Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) &= (1.01 \pm 0.11) \cdot 10^{-10} & Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= (2.7 \pm 0.3) \cdot 10^{-11} \end{aligned} \quad (42)$$

The curve "superweak" in fig. 3 is the ambiguity curve of Winstein [50]. If $(\bar{\varrho}, \bar{\eta})$ lies on this curve it is impossible to distinguish the standard model from superweak models on the basis of CP-asymmetries in neutral B-decays to CP-eigenstates. It is evident that in order to make this distinction both $\sin(2\alpha)$ and $\sin(2\beta)$ have to be measured.

6.4 ε_K , $B_d^o - \bar{B}_d^o$ Mixing, $\sin(2\beta)$ and $\sin(2\alpha)$

We next combine the analysis of sections 6.1 and 6.3. In fig. 4 we show the allowed ranges for $\sin(2\alpha)$ and $\sin(2\beta)$ corresponding to the ranges I-III in (36)–(38) and the range IV, defined in [4], together with the results of the independent measurements of $\sin(2\beta) = 0.60 \pm 0.06$ and $\sin(2\alpha)$ of (41). The latter are represented by dark shaded rectangles. The black rectangles illustrate the accuracy ($\Delta \sin(2\alpha) = \pm 0.04$, $\Delta \sin(2\beta) = \pm 0.02$) to be expected from B-physics at Fermilab during the Main Injector era [51] and the first phase of LHC [52].

The impact of the measurements of $\sin(2\beta)$ and $\sin(2\alpha)$ is clearly visible on this plot. In cases III and IV we have examples where the measurements of $\sin(2\alpha)$ are incompatible with the predictions coming from ε_K and $B^o - \bar{B}^o$ mixing. This would be a signal for a physics beyond the standard model. The measurement of $\sin(2\alpha)$ is essential for this. Analogous comments apply to the exclusion of superweak models.

7 Precise Determinations of the CKM Matrix

Let us finally concentrate on the decays of class I which being essentially free from any hadronic uncertainties, stand out as ideally suited for the determination of the CKM parameters. We will use as inputs [47]:

- $|V_{us}| \equiv \lambda = 0.2205 \pm 0.0018$ determined in [53, 54]. Recent critical discussions of this determination and of the related element $|V_{ud}|$ can be found in [55].
- $a \equiv \sin(2\alpha)$, $b \equiv \sin(2\beta)$ to be measured in future B-physics experiments.
- $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ to be measured hopefully one day at Fermilab (KAMI), KEK or another laboratory.

Using (32) and (24) one determines ϱ , η and $|V_{cb}|$ as follows [47]:

$$\bar{\varrho} = 1 - \bar{\eta} r_+(b) \quad , \quad \bar{\eta} = \frac{r_-(a) + r_+(b)}{1 + r_+^2(b)} \quad (43)$$

$$|V_{cb}| = \lambda^2 \left[\frac{\sqrt{B_L}}{\eta X(x_t)} \right]^{1/2} \quad B_L = \frac{Br(K_L \rightarrow \pi^0 \nu \bar{\nu})}{1.94 \cdot 10^{-10}} \quad (44)$$

ϱ and η is to be found from (6) and (43). Here we have introduced

$$r_{\pm}(z) = \frac{1}{z} (1 \pm \sqrt{1 - z^2}) \quad z = a, b \quad (45)$$

Note that the factor in front of λ^2 in (44) gives the parameter A in the Wolfenstein parametrization. Using (25) we also find a useful formula

$$|V_{cb}| = 39.1 \cdot 10^{-3} \sqrt{\frac{0.39}{\eta}} \left[\frac{170 \text{ GeV}}{m_t} \right]^{0.575} \left[\frac{Br(K_L \rightarrow \pi^0 \nu \bar{\nu})}{3 \cdot 10^{-11}} \right]^{1/4} \quad (46)$$

We note that the weak dependence of $|V_{cb}|$ on $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ allows to achieve high accuracy for this CKM element even when $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is known within 5 – 10% accuracy. There exist other solutions for ϱ and η coming from (32). As shown in [47] they can all be eliminated on the basis of the present knowledge of the CKM matrix.

At first sight it is probably surprising that we use a rare K-meson decay to determine $|V_{cb}|$. The natural place to do this are of course tree level B-decays. On the other hand using unitarity and the Wolfenstein parametrization with $|V_{cb}| = A\lambda^2$ it is clear that $|V_{cb}|$ gives the overall scale A of the top quark couplings V_{td} and V_{ts} which are the only CKM couplings in $K_L \rightarrow \pi^0 \nu \bar{\nu}$. From this point of view it is very natural to measure the parameter A in a short distance process involving the top quark and using unitarity of the CKM matrix to find the value of $|V_{cb}|$. Moreover this strategy, in contrast to tree-level B-decays, is free from hadronic uncertainties. On the other hand one should keep in mind that this method contains the uncertainty from the physics beyond the standard model which could contribute to short distance processes like $K_L \rightarrow \pi^0 \nu \bar{\nu}$. We will return to this below.

As illustrative examples, let us consider the following three scenarios:

Scenario I

$$\begin{aligned} \sin(2\alpha) &= 0.40 \pm 0.08 & \sin(2\beta) &= 0.70 \pm 0.06 \\ Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= (3.0 \pm 0.3) \cdot 10^{-11} & m_t &= (170 \pm 5) \text{ GeV} \end{aligned} \quad (47)$$

Scenario II

$$\begin{aligned} \sin(2\alpha) &= 0.40 \pm 0.04 & \sin(2\beta) &= 0.70 \pm 0.02 \\ Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= (3.00 \pm 0.15) \cdot 10^{-11} & m_t &= (170 \pm 3) \text{ GeV} \end{aligned} \quad (48)$$

Scenario III

$$\begin{aligned} \sin(2\alpha) &= 0.40 \pm 0.02 & \sin(2\beta) &= 0.70 \pm 0.01 \\ Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= (3.00 \pm 0.15) \cdot 10^{-11} & m_t &= (170 \pm 3) \text{ GeV} \end{aligned} \quad (49)$$

The accuracy in the scenario I should be achieved at B-factories [48], HERA-B [49], at KAMI [56] and at KEK [57]. Scenarios II and III correspond to B-physics at Fermilab during the Main Injector era [51] and at LHC [52] respectively. At that time an improved measurement of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ should be aimed for.

	Central	<i>I</i>	<i>II</i>	<i>III</i>
ϱ	0.072	± 0.040	± 0.016	± 0.008
η	0.389	± 0.044	± 0.016	± 0.008
R_t	1.004	± 0.025	± 0.012	± 0.006
$ V_{cb} /10^{-3}$	39.2	± 3.9	± 1.7	± 1.3
$ V_{ub}/V_{cb} $	0.087	± 0.010	± 0.003	± 0.002
$ V_{td} /10^{-3}$	8.7	± 0.9	± 0.4	± 0.3
$ V_{ts} /10^{-3}$	38.4	± 3.8	± 1.7	± 1.3
$\sin(2\gamma)$	0.35	± 0.15	± 0.07	± 0.04
B_K	0.83	± 0.17	± 0.07	± 0.06
$F_B\sqrt{B_B}$	200	± 19	± 8	± 6

Table 1: Determinations of various parameters in scenarios I-III [47]

The results that would be obtained in these scenarios for ϱ , η , R_t , $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $|V_{td}|$, $|V_{ts}|$ and $\sin(2\gamma)$ are collected in table 1.

Table 1 shows very clearly the potential of CP asymmetries in B-decays and of $K_L \rightarrow \pi^0\nu\bar{\nu}$ in the determination of CKM parameters. It should be stressed that this high accuracy is not only achieved because of our assumptions about future experimental errors in the scenarios considered, but also because $\sin(2\alpha)$ is a very sensitive function of ϱ and η [4], $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$ depends strongly on $|V_{cb}|$ and most importantly because of the clean character of the quantities considered.

In table 1 we have also shown the values of the non-perturbative parameters B_K and $F_B\sqrt{B_B}$ (in MeV) which can be extracted from the data on ε_K and $B_d^o - \bar{B}_d^o$ mixing once the CKM parameters have been determined in the scenarios considered. To this end $x_d = 0.72$ and $\tau(B) = 1.5$ ps have been assumed. The errors on these two quantities should be negligible at the end of this millennium. Note that the resulting central values for B_K in table 1 are close to the lattice [13] results. Similar patterns of uncertainties emerge for different central input parameters [47].

It is instructive to investigate whether the use of $K^+ \rightarrow \pi^+\nu\bar{\nu}$ instead of $K_L \rightarrow \pi^0\nu\bar{\nu}$ would also give interesting results for V_{cb} and V_{td} . We again consider scenarios I-III with $Br(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10}$ for the scenario I and $Br(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (1.0 \pm 0.05) \cdot 10^{-10}$ for scenarios II and III in place of $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$ with all other input parameters unchanged. An analytic formula for $|V_{cb}|$ can be found in [47]. The results for ϱ , η , R_t , $|V_{ub}/V_{cb}|$ and $\sin(2\gamma)$ remain of course unchanged. In table 2 we

	Central	<i>I</i>	<i>II</i>	<i>III</i>
$ V_{cb} /10^{-3}$	41.2	± 4.3 (3.2)	± 3.0 (1.9)	± 2.8 (1.8)
$ V_{td} /10^{-3}$	9.1	± 0.9 (0.7)	± 0.6 (0.4)	± 0.6 (0.4)
$F_B\sqrt{B_B}$	190	± 17 (12)	± 12 (7)	± 12 (7)

Table 2: Determinations of various parameters in scenarios I-III using $K^+ \rightarrow \pi^+\nu\bar{\nu}$ instead of $K_L \rightarrow \pi^0\nu\bar{\nu}$. The values in parentheses show the situation when the uncertainties in m_c and $\Lambda_{\overline{MS}}$ are not included.

show the results for $|V_{cb}|$, $|V_{td}|$ and $F_B\sqrt{B_B}$. We observe that due to the uncertainties present in the charm contribution to $K^+ \rightarrow \pi^+\nu\bar{\nu}$, which was absent in $K_L \rightarrow \pi^0\nu\bar{\nu}$, the determinations of $|V_{cb}|$, $|V_{td}|$ and $F_B\sqrt{B_B}$ are less accurate. If the uncertainties due to the charm mass and $\Lambda_{\overline{MS}}$ are removed one day, only the uncertainty related to μ will remain in $P_0(K^+)$ giving $\Delta P_0(K^+) = \pm 0.03$ [37]. In this case the results in parentheses in table 2 would be found.

To summarize we have seen that the measurements of the CP asymmetries in neutral B-decays together with a measurement of $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$ and the known value of $|V_{us}|$ offer a precise determination of *all* elements of the Cabibbo-Kobayashi-Maskawa matrix essentially without any hadronic uncertainties. $K_L \rightarrow \pi^0\nu\bar{\nu}$ proceeds almost entirely through direct CP violation and is known to be a very useful decay for the determination of η . However due to the strong dependence on V_{cb} this determination cannot fully compete with the one which can be achieved using CP asymmetries in B-decays. As the analysis of [41] shows (see section 6.2) it will be difficult to reach $\Delta\eta = \pm 0.03$ this way if $|V_{cb}|$ is determined in tree level B-decays. Our strategy then is to find η from CP asymmetries in B decays and use $K_L \rightarrow \pi^0\nu\bar{\nu}$ to determine $|V_{cb}|$. To our knowledge no other decay can determine $|V_{cb}|$ as cleanly as this one.

We believe that the strategy presented in [47] is the theoretically cleanest way to establish the precise values of the CKM parameters. The quantities of class II are also theoretically rather clean and are useful in this respect but they cannot compete with the quantities of class I considered here (see our remarks in section 5.2 and in [47]). On the other hand once ϱ , η and $|V_{cb}|$ (or A) have been precisely determined as discussed here, it is clear that x_d/x_s , $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ and Δ_{LR} can be rather accurately predicted and confronted with future experimental data. Such confrontations would offer excellent tests of the standard model and could possibly give signs of a new physics beyond it.

Of particular interest will also be the comparison of $|V_{cb}|$ determined as suggested here with the value of this CKM element extracted from tree level semi-leptonic B-decays [6, 8]. Since in contrast to $K_L \rightarrow \pi^0 \nu \bar{\nu}$, the tree-level decays are to an excellent approximation insensitive to any new physics contributions from very high energy scales, the comparison of these two determinations of $|V_{cb}|$ would be a good test of the standard model and of a possible physics beyond it. Also the values of $|V_{ub}/V_{cb}|$ from tree-level B-decays, which are subject to hadronic uncertainties larger than in the case of V_{cb} , when compared with the clean determinations suggested here could teach us about the reliability of non-perturbative methods. The same applies to the quantities like x_d and the CP violating parameter ε_K which are subject to uncertainties present in the non-perturbative parameters $F_B \sqrt{B_B}$ and B_K respectively.

It is also clear that once the accuracy for CKM parameters presented here has been attained, also detailed tests of proposed schemes for quark matrices [58, 59] will be possible.

Precise determinations of all CKM parameters without hadronic uncertainties along the lines suggested here can only be realized if the measurements of CP asymmetries in B-decays and the measurements of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ can reach the desired accuracy. All efforts should be made to achieve this goal.

8 Final Remarks

In this review we have discussed the most interesting quantities which when measured should have important impact on our understanding of the CP violation and of the quark mixing. We have discussed both CP violating and CP conserving loop induced decays because in the standard model CP violation and quark mixing are closely related.

In this short review we have concentrated on the CP violation in the standard model. The structure of CP violation in extensions of the standard model could deviate from this picture [9, 60]. Consequently the situation in this field could turn out to be very different from the one presented here. Unfortunately in these extensions new parameters appear and a quantitative analysis of CP violation is more difficult. The charm meson decays could turn out to be a very good place to look for new physics effects.

Although the search for the unitarity triangle and the tests of the Kobayashi-Maskawa picture of CP violation is an important target of particle physics, we should not forget that what we are really after is the true origin of CP violation observed in nature. The strategies presented here may shed some light in which direction we should go. However simply finding the values of ϱ and η or demonstrating that the KM picture

of CP violation is correct or false is certainly and fortunately not the whole story. In order to understand the true origin of CP violation in nature we need new experiments and new theoretical ideas.

I would like to thank the organizers, in particular Prof. G. Martinelli, Prof. Diambri-Palazzi and Prof. L. Zanello for such a pleasant atmosphere during this symposium. I would also like to thank all my collaborators for exciting time we had together and Gerhard Buchalla, Robert Fleischer and Mikolaj Misiak for a critical reading of the manuscript.

References

- [1] N. Cabibbo, *Phys. Rev. Lett.* **10** (1963) 531.
- [2] M. Kobayashi and K. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.
- [3] L. Wolfenstein, *Phys. Rev. Lett.* **51** (1983) 1945.
- [4] A.J. Buras, M.E. Lautenbacher and G. Ostermaier, MPI-Ph/94-14, TUM-T31-57/94.
- [5] Particle Data Group, *Phys. Rev.* **D 45** (1992), No.11 part II.
- [6] S. Stone, in B Decays II ed. by S. Stone, World Scientific; HEPSY 93-11.
- [7] C. Jarlskog and R. Stora, *Phys. Lett.* **B 208** (1988) 268.
- [8] M. Neubert, *Phys. Lett.* **B 264** (1991) 455; P. Ball, *Phys. Lett.* **B 281** (1992) 133; M. Luke and M. Savage, *Phys. Lett.* **B 321** (1994) 88; M. Shifman, N.G. Uraltsev, A. Vainshtein, TPI-MINN-94/13-T; I. Bigi and N. Uraltsev, CERN-TH.7063/93; M. Beneke and V.M. Braun, MPI-PhT/94-9; P. Ball and U. Nierste, TUM-T31-56R/94; T. Mannel and M. Neubert, CERN-TH.7156/94; M. Neubert, CERN-TH.7225/94.
- [9] Y. Nir and H.R. Quinn in " B Decays ", ed S. Stone (World Scientific, 1992), p. 362; I. Dunietz, *ibid* p.393 and refs. therein.
- [10] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, *Phys. Rev. Lett.* **13** (1964) 138.
- [11] A.J. Buras, *Phys. Lett.* **B 317** (1993) 449.
- [12] F. Abe et al. (CDF Collaboration) FERMI-PUB-94/097-E.
- [13] G.W. Kilcup, S.R. Sharpe, and R. Gupta, UW/PT-93-25.
- [14] W. A. Bardeen, A.J. Buras, and J.-M. Gérard, *Phys. Lett.* **B 211** (1988) 343.

- [15] M. Savage and M. Wise, *Phys. Lett.* **B 250** (1990) 151; M. Lu, M. Wise and M. Savage, *Phys. Rev.* **D 46** (1992) 5026; G. Belanger, C.Q. Geng and P. Turcotte, *Nucl. Phys.* **B 390** (1993) 253.
- [16] G. Buchalla and A.J. Buras, TUM-T31-65/94.
- [17] F.J. Gilman and M.B. Wise, *Phys. Lett.* **B 83** (1979) 83; J. Bijmens and M.B. Wise, *Phys. Lett.* **B 137** (1984) 245.
- [18] J.M. Flynn and L. Randall, *Phys. Lett.* **B 224** (1989) 221.
- [19] G. Buchalla, A.J. Buras and M.K. Harlander, *Nucl. Phys.* **B 337** (1990) 313.
- [20] A.J. Buras, M.Jamin and M.E. Lautenbacher, *Nucl. Phys.* **B 408** (1993) 209.
- [21] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Phys. Lett.* **B 301** (1993) 263.
- [22] A.J. Buras and M.E. Lautenbacher, *Phys. Lett.* **B 318** (1993) 212.
- [23] J. Heinrich, E.A. Paschos, J.-M. Schwarz, and Y. L. Wu, *Phys. Lett.* **B 279** (1992) 140.
- [24] G.D. Barr et al. *Phys. Lett.* **B 317** (1993) 233.
- [25] L.K. Gibbons et al., *Phys. Rev. Lett.* **70** (1993) 1203.
- [26] L. Wolfenstein, *Phys. Rev. Lett.* **13** (1964) 562.
- [27] F.J. Gilman and M.B. Wise, *Phys. Rev.* **D 20** (1979) 934; *Phys. Rev.* **D 21** (1980) 3150. G. Ecker, A. Pich and E. de Rafael, *Nucl. Phys.* **B 303** (1988) 665.
- [28] A.G. Cohen, G. Ecker, and A. Pich, *Phys. Lett.* **B 304** (1993) 347; A. Pich, CERN-Th-7114-93 and references therein.
- [29] G.D. Barr et al. *Phys. Lett.* **B 304** (1993) 381.
- [30] A.J. Buras, M.E. Lautenbacher, M. Misiak and M. Münz, TUM-T31-60/94.
- [31] C.O. Dib, I. Dunietz, and F.J. Gilman, *Phys. Lett.* **B 218** (1989) 487; *Phys. Rev.* **D 39** (1989) 2639; J.M. Flynn and L. Randall, *Nucl. Phys.* **B 326** (1989) 31.
- [32] D.A Harris et al., *Phys. Rev. Lett.* **71** (1993) 3918.
- [33] K.E. Ohl et al., *Phys. Rev. Lett.* **64** (1990) 2755.
- [34] L. Littenberg and G. Valencia, *Ann. Rev. Nucl. Part. Sci.* **43** (1993) 729; B. Winstein and L. Wolfenstein, *Rev. Mod. Phys.* **65** (1993) 1113; J.L. Ritchie and S.G. Wojcicki, *Rev. Mod. Phys.* **65** (1993) 1149.
- [35] L.S. Littenberg, *Phys. Rev.* **D 39** (1989) 3322.

- [36] G. Buchalla and A.J. Buras, *Nucl. Phys.* **B 400** (1993) 225.
- [37] G. Buchalla and A.J. Buras, *Nucl. Phys.* **B 412** (1994) 106.
- [38] C.O. Dib, I. Dunietz and F.J. Gilman, *Mod. Phys. Lett.* **A 6** (1991) 3573.
- [39] D. Rein and L.M. Sehgal, *Phys. Rev.* **D 39** (1989) 3325; J.S. Hagelin and L.S. Littenberg, *Prog. Part. Nucl. Phys.* **23** (1989) 1; M. Lu and M.B. Wise, CALT-68-1911 (1994).
- [40] A.J. Buras and M.K. Harlander, A Top Quark Story, *in Heavy Flavors*, eds. A.J. Buras and M. Lindner, World Scientific (1992), p.58.
- [41] G. Buchalla and A.J. Buras, MPI-PhT/94-19, TUM-T31-59/94.
- [42] M. Gronau and D. London, *Phys. Rev. Lett.* **65** (1990) 3381, *Phys. Lett.* **B 253** (1991) 483; Y. Nir and H. Quinn, *Phys. Rev.* **D 42** (1990) 1473, *Phys. Rev. Lett.* **67** (1991) 541; R. Aleksan, I. Dunietz, B. Kayser and F. Le Diberder, *Nucl. Phys.* **B 361** (1991) 141.
- [43] R. Aleksan, B. Kayser and D. London, NSF-PT-93-4, NSF-PT-94-2 and refs. therein; R. Fleischer, TUM-T31-58/94 (*Phys. Lett.* 1994).
- [44] M. Gronau and D. Wyler, *Phys. Lett.* **B 265** (1991) 172.
- [45] A.J. Buras, M. Jamin, and P.H. Weisz, *Nucl. Phys.* **B 347** (1990) 491.
- [46] B. Grinstein et al., *Nucl. Phys.* **B 380** (1992) 369.
- [47] A.J. Buras, MPI-PhT/94-25, TUM-T31-61/94; hep-ph 9405368.
- [48] BaBar Collaboration – Status report, SLAC-419 (June 1993), BELLE Collaboration (KEKB) -KEK-94-2 (April 1994).
- [49] H. Albrecht et al. (HERA-B), DESY-PRC 92/04 (1992).
- [50] B. Winstein, *Phys. Rev. Lett.* **68** (1992) 1271.
- [51] CDF Collaboration and Others, CDF/DOC/ADVISORY/PUBLIC/2436 (May 1994); R. Edelstein et al. EOI (May 1994).
- [52] S. Erhan et al. (COBEX), *Nucl. Instr. and Methods* **A333** (1993) 101; ATLAS Collaboration, CERN/LHCC/93-53; Workshop on Beauty 93, Liblice Castle, Czech Republic, 1993, ed. P. Schlein, *Nucl. Instr. and Methods* **A333** (1993) 1-230.
- [53] H. Leutwyler and M. Roos, *Zeitschr. f. Physik* **C25** (1984) 91.
- [54] J.F. Donoghue, B.R. Holstein and S.W. Klimt, *Phys. Rev.* **D 35** (1987) 934.
- [55] W.J. Marciano, *Annu. Rev. Nucl. Part. Sci.* **41** (1991) 469 and talk given at the Symposium on Heavy Flavours, Montreal, Canada, 1993.

- [56] K. Arisaka et al., KAMI conceptual design report, FNAL, June 1991.
At present $Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 5.8 \cdot 10^{-5}$ (M. Weaver et. al (FNAL E799), Phys. Rev. Lett. to appear.)
- [57] T. Inagaki, T. Sato and T. Shinkawa, Experiment to search for the decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ at KEK 12*GeV* Proton Synchrotron, 30 Nov. 1991.
- [58] S. Dimopoulos, L.J. Hall and S. Raby, *Phys. Rev.* **D 45** (1992) 4192.
- [59] P. Ramond, R.G Roberts and G.G. Ross, *Nucl. Phys.* **B 406** (1993) 19.
- [60] L.J. Hall and S. Weinberg, *Phys. Rev.* **D 48** (1993) 979.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9406272v1>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9406272v1>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9406272v1>